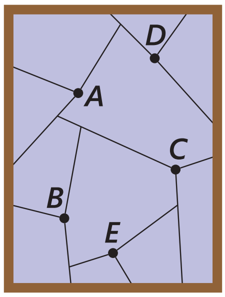
3 points problems

01. The year 2025 is a perfect square because $2025 = 45^2$. How many years will pass until the next year which is a perfect square?

(A) 25 **(B)** 91 **(C)** 121 **(D)** 500 **(E)** 2025

02. During a hailstorm, five hailstones hit a glass window, one after the other, at points ***A***, ***B***, ***C***, ***D*** and ***E***, though not necessarily in that order. Where each hailstone hits the glass, it creates some linear cracks which stop either at a previous crack or at the boundary.



In which order did the hailstones hit the glass?

(A) *DACBE* **(B)** *ABCDE* **(C)** *BDACE* **(D)** *BCDAE* **(E)** *DCABE*

03. Vasily has 20 different coloured balls either yellow or green or blue or black. Of these, exactly 17 are not green, 15 are not black, and 12 are not yellow.

How many balls are blue?

(A) 8 **(B)** 7 **(C)** 6 **(D)** 4 **(E)** 3

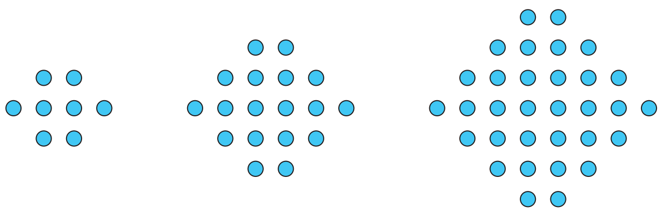
04. In what interval does the value of 88×888 lie?

(A) Between 8 and 88.
(B) Between 88 and 888.
(C) Between 888 and 8 888.
(D) Between 8 888 and 88 888.
(E) Between 88 888 and 888 888.

05. Which of the following is equal to the square root of 16^{16}?

(A) 4^4 **(B)** 4^8 **(C)** 4^{16} **(D)** 8^8 **(E)** 16^4

06. The shapes shown below are the first three shapes of a sequence. How many dots make up the fifth shape in the sequence?



(A) 72 **(B)** 74 **(C)** 76 **(D)** 78 **(E)** 80

07. Mike obtains a number $\boldsymbol{x}$ by dividing the number $\sqrt{11}$ by three. Where is the number $\boldsymbol{x}$ to be found on the number line?

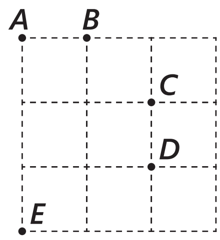
(A) Between 0 and 1.
(B) Between 1 and 2.
(C) Between 2 and 3.
(D) Between 3 and 4.
(E) Between 4 and 5.

08. Silia's favourite chocolate bars come in packets. Each packet used to contain five bars. Now they contain only four but are sold at the same price.

By what percentage has the price of each bar increased?

(A) By 10% **(B)** By 20% **(C)** By 25% **(D)** By 30% **(E)** By 50%

09. Robert wants to choose four points so that the distances between each pair of points are different.



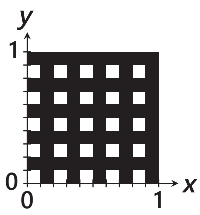
Which of the points ***A***, ***B***, ***C***, ***D*** and ***E*** should be removed?

(A) *A* **(B)** *B* **(C)** *C* **(D)** *D* **(E)** *E*

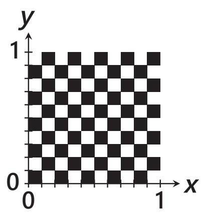
10. In the ***xy***-plane, some parts of the square defined by $0 \le x \le 1$, $0 \le y \le 1$ were painted black. A point (x, y) lies in one of these black parts if and only if for both ***x*** and ***y*** the first digit after the decimal point is odd.

How does the square look after it has been painted?

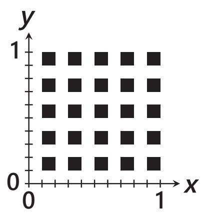
(A)



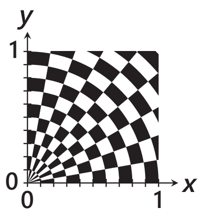
(C)



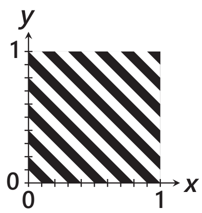
(E)



(B)

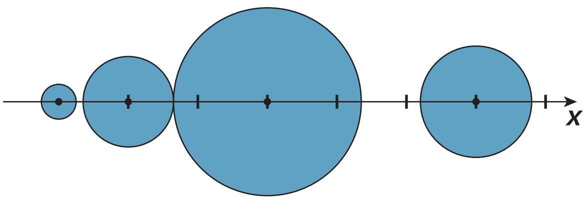


(D)



4 points problems

11. Four circular discs with positive radii r_1, r_2, r_3 and r_4 are centred at (0; 0), (1; 0), (3; 0) and (6; 0). The discs may touch but not overlap.



What is the largest possible value of $r_1 + r_2 + r_3 + r_4$?

(A) 3

(B) 4

(C) 5

(D) 6

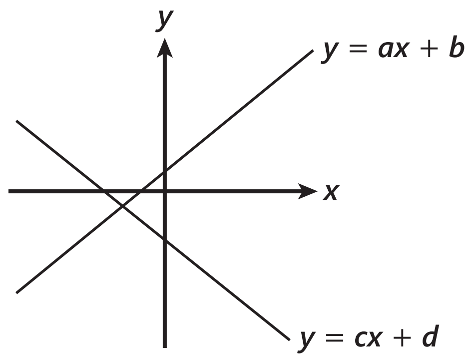
(E) There is no upper limit.

12. Amongst 10 different positive integers, there are exactly five that are divisible by 5 and exactly seven that are divisible by 7. Let M be the largest of these numbers.

What is the smallest possible value of M?

(A) 105 **(B)** 77 **(C)** 75 **(D)** 70 **(E)** 63

13. A student drew the graphs of two linear functions in a coordinate system, as shown.



Which statement is always true about the expression $ab + cd - (ac + bd)$?

(A) It is negative.

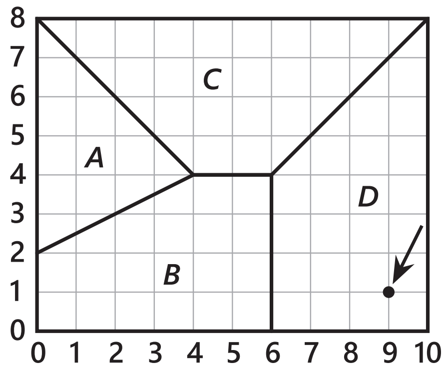
(B) It is non-positive.

(C) It is positive.

(D) It is zero.

(E) None of the others is always true.

14. The map shows a small town which has 4 schools. The map shows the regions A, B, C and D of all the points nearest, respectively, to each school. The coordinates of the school in region D are (9, 1).



What are the coordinates of the school in region A?

(A) (0, 4)

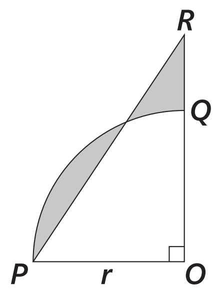
(B) (1, 4)

(C) (1, 5)

(D) (1, 6)

(E) (2, 4)

15. The diagram shows a quarter-circle OPQ of radius r and a triangle OPR. The two shaded regions have the same area.



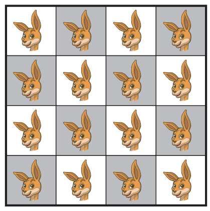
What is the length of OR?

(A) $\frac{\pi r}{2}$ **(B)** $\frac{3r}{2}$ **(C)** πr **(D)** $\frac{2}{\pi}$ **(E)** $\frac{\pi}{2r}$

16. What is the smallest positive integer N such that $\sqrt{2\sqrt{3\sqrt{N}}}$ is an integer?

(A) $2^{12} \cdot 3^{6}$ **(B)** $2^{4} \cdot 3^{14}$ **(C)** $2^{4} \cdot 3^{6} \cdot 5^{8}$ **(D)** $2^{4} \cdot 3^{2}$ **(E)** $2^{4} \cdot 3^{6}$

17. On a 4×4 giant chessboard there are 16 kangaroos, one in each square. On each turn, each of the kangaroos jumps to a neighbouring square (up, down, left or right, but not diagonally). All kangaroos stay on the board. There can be several kangaroos on any square.



After 100 turns, what is the largest possible number of empty squares?

(A) 15 **(B)** 14 **(C)** 12 **(D)** 10 **(E)** 8

18. The five digit number ***N18NN*** is divisible by 18. Which of the following statements is true about the digit ***N***?

(A) There is exactly one such ***N***.

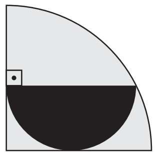
(B) There are exactly two such ***N***.

(C) There are exactly three such ***N***.

(D) There are more than three such ***N***.

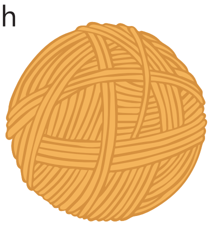
(E) No such ***N*** exists.

19. The area of the black semicircle is 12 cm². What is the area of the big quarter circle?



(A) 42 cm² **(B)** 36 cm² **(C)** 32 cm² **(D)** 30 cm² **(E)** 25 cm²

20. When grandma started knitting woollen socks, she had a huge ball of yarn with a diameter of 30 cm. After finishing 70 socks, she still has a ball of yarn with a diameter of 15 cm.

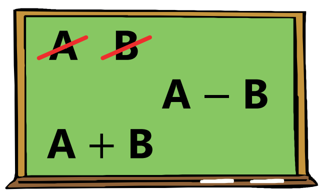


How many more socks can grandma knit with the yarn that is left?

(A) 70 **(B)** 50 **(C)** 30 **(D)** 20 **(E)** 10

5 points problems

21. A student starts with two numbers written on the board. He then deletes them and writes the sum of the numbers and the positive difference of the numbers. He continues the same process with the new numbers. He starts with the numbers 3 and 5 and repeats the process 50 times.



What are the two numbers he will end up with?

(A) 3^{25} and 5^{25}

(B) 3^{50} and 5^{50}

(C) $2 \cdot 3^{25}$ and $2 \cdot 5^{25}$

(D) $3 \cdot 2^{25}$ and $5 \cdot 2^{25}$

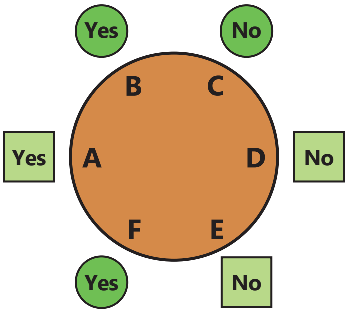
(E) None of the previous.

22. John wrote an arbitrary two-digit integer on a blackboard. Then, he erased the last digit of the integer. As a result, the initial integer decreased by ***p***%.

Which of the following is the closest to the largest possible value of ***p***?

(A) 10 **(B)** 50 **(C)** 90 **(D)** 95 **(E)** 99

23. A group of three square men from Mars and a group of three circular men from Jupiter are sitting around a table, as shown. One of the six has the key to their flying saucer. All members of one group always tell the truth and all members of the other group always lie. All six were asked the question "does a person sitting next to you have the key?". Their answers are shown in the figure.



Who has the key?

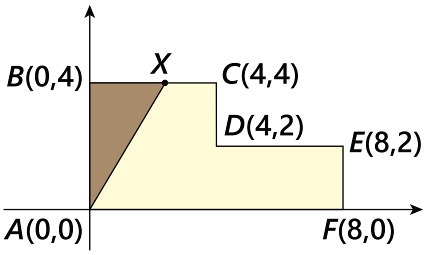
(A) A **(B)** B **(C)** C **(D)** D **(E)** E

24. Julia and her little sister Paula go out together for a bike ride. Both ride at a constant speed: Julia at 18 km/h and Paula at 12 km/h and they follow the same path. Julia feels tired after 20 minutes and decides to go back. When she meets Paula, Julia tells her to turn around and both return home, each at their own speed.

How many minutes later than Julia will Paula arrive home?

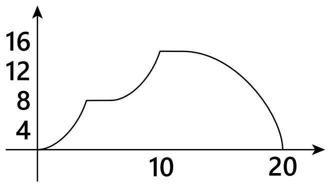
(A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 15

25. The point X moves from point B to point A clockwise along the sides of the polygonal line $BCDEFA$ represented on the cartesian plane, as shown. Let $f(x)$ be the area of the triangle ABX such that x is the distance covered by the point X.

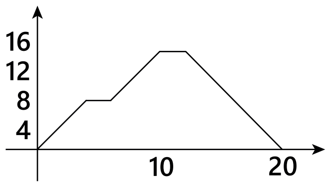


What is the aspect of the graph of the function f?

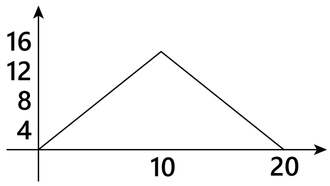
(A)



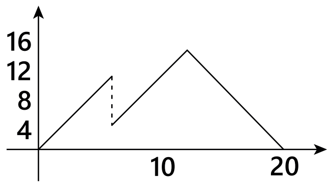
(B)



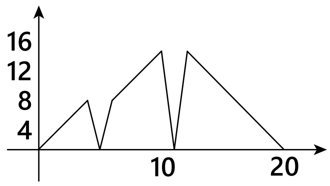
(C)



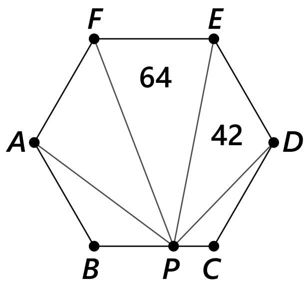
(D)



(E)



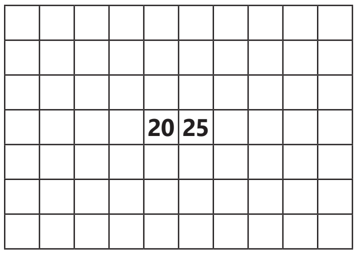
26. The diagram shows a regular hexagon $ABCDEF$. Point P lies on BC so that the area of PEF is 64 and the area of PDE is 42.



What is the area of APF?

(A) 53 **(B)** 54 **(C)** 56 **(D)** 60 **(E)** 64

27. Patricia has written a number in each cell of a 7×10 table. The sum of all the numbers in any 3×4 or 4×3 rectangle is zero. The numbers in two of the cells are shown in the diagram.



What is the sum of all the numbers in the table?

(A) -5

(B) -20

(C) -25

(D) -45

(E) It is not possible to determine.

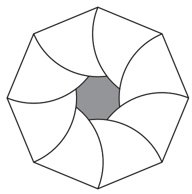
28. Three boxes contain three balls each. The inscriptions on the labels show the contents of each box. The labels are rearranged so that none of them shows the contents correctly.

Mike picks a box, randomly removes a ball from it, and notes its colour without putting it back.

What is the smallest number of balls that Mike needs to remove to determine the contents of each box?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

29. The figure shows a regular octagon of side 1 cm. An arc of radius 1 cm has been drawn centred at each vertex, as shown.



What is the perimeter of the shaded region?

(A) π cm

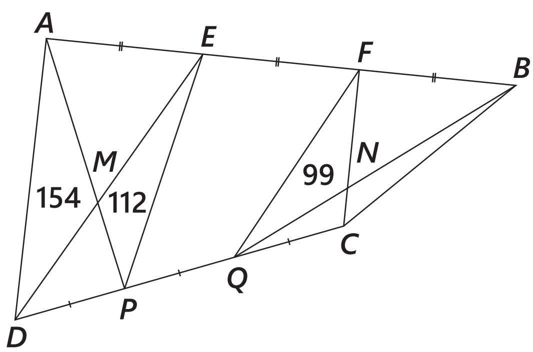
(B) $\frac{2\pi}{3}$ cm

(C) $\frac{8\pi}{9}$ cm

(D) $\frac{4\pi}{5}$ cm

(E) $\frac{3\pi}{4}$ cm

30. The sides AB and CD of the convex quadrilateral $ABCD$ are each divided into three parts by points E, F, P and Q so that $AE = EF = FB$ and $DP = PQ = QC$. The diagonals of $AEPD$ and $FBCQ$ intersect at M and N respectively. The areas of triangles AMD, EMP and FNQ are 154, 112 and 99 respectively.



What is the area of triangle BCN?

(A) 57

(B) 70

(C) 72

(D) 86

(E) 141